Quantization of the Relativistic Particle

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Abstract

We revise the problem of the quantization of relativistic particle, presenting a modified consistent canonical scheme, which allows one not only to include arbitrary backgrounds in the consideration but to get in course of the quantization a consistent relativistic quantum mechanics, which reproduces literally the behavior of the one-particle sector of the corresponding quantum field. At the same time this construction presents a possible solution of the well-known old problem how to construct a consistent quantum mechanics on the base of a relativistic wave equation.

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Already for a long time there exists a definite interest in quantization of classical and pseudoclassical models of relativistic particles (RP). This problem meets such difficulties as zero-Hamiltonian phenomenon and time definition problem. Consideration in arbitrary electromagnetic and gravitational backgrounds creates additional difficulties. The usual aim of the quantization is to arrive in a way to a corresponding relativistic wave equation without any attempt to demonstrate that a consistent quantum mechanics is constructed, since there is a common opinion that the construction of a such a mechanics on the base of relativistic wave equations is not possible due to existence of infinite number of negative energy levels, and due to existence of negative vector norms (in scalar case), and these difficulties may be only solved in QFT [1]. One of possible approach to the canonical quantization of RP models was presented in [2] on the base of a special gauge, which fixes reparametrization gauge freedom. However, the difficulties with inclusion of arbitrary backgrounds were not overcome and the consistent quantum mechanics was not constructed. It turns out that the whole scheme of quantization, which was used in that papers and repeated then in numerous works, has to be changed essentially to make it possible to solve the above problems and to construct a quantum mechanics which is consistent to the same extent to which a one-particle description is possible in the frame of the corresponding QFT. One of the main point of the modification is related to a principally new realization of the Hilbert space. At the same time this construction gives a solution of the above mentioned old problem how to construct a consistent quantum mechanics on the base of a relativistic wave equation Below we present a demonstration for a spinless particle case. The spinning particle case and all long technical details may be found by a reader in [3].

We start with a reparametrization invariant action of a spinless relativistic particle interacting with gravitational and electromagnetic backgrounds,

$$S=\int_0^1 L d\tau,\ L=-m\sqrt{\dot{x}^\mu g_{\mu\nu}(x)\dot{x}^\nu}-q\dot{x}^\mu g_{\mu\nu}(x)A^\nu(x),\ \dot{x}^\mu=dx^\mu/d\tau. \tag{1}$$

We select a special gauge $g_{0i}=0$ (then $g^{00}=g_{00}^{-1}>0,\ g^{ik}g_{kj}=\delta^i_j$) of the metric and define canonical momenta

$$p_\mu = \frac{\partial L}{\partial \dot{x}^\mu} = -\frac{m g_{\mu\nu}\dot{x}^\nu}{\sqrt{\dot{x}^2}} - qA_\mu\ ,\quad A_\mu = g_{\mu\nu}A^\nu . \tag{2}$$

The discrete variable $\zeta = \pm 1$ is important for our consideration,

$$\zeta = -\mathrm{sign}\left[p_0 + qA_0\right] . \tag{3}$$

It follows from (2): $\mathrm{sign}(\dot{x}^0) = \zeta$, and there is a constraint $\Phi_1 = p_0 + qA_0 + \zeta\omega = 0$. The total Hamiltonian $H^{(1)}$ we construct according to a standard procedure [4],

$$\dot{\eta} = \{\eta, H^{(1)}\},\ \Phi_1 = 0,\ \lambda > 0,\ \ H^{(1)} = \zeta\lambda\Phi_1,\ \eta = (x^\mu, p_\mu),\ (\lambda = |\dot{x}^0|). \tag{4}$$

$\Phi_1 = 0$ is a first-class constraint. A possible gauge condition, which fixes only λ, has the form [2]:

$$\Phi_2 = x^0 - \zeta\tau = 0\ . \tag{5}$$

We study equations of motion to clarify the meaning of ζ (below for simplicity $g_{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}(1, -1, \ldots, -1)$). Interpreting $\zeta\tau = x^0$ as a physical time, $\zeta p_i = P_i$ as a physical momentum, $dx^j/d(\zeta\tau) = dx^j/dx^0 = v^j$ as a physical three-velocity, $\mathcal{P}_i^{kin} = P_i + (\zeta q)A_i$ as the kinetic momentum, we may see that (4) in the gauge (5) read:

$$\frac{d\boldsymbol{\mathcal{P}}_{kin}}{dx^0} = (\zeta q)\left\{\mathbf{E} + [\mathbf{v}, \mathbf{H}]\right\},\ \mathbf{v} = \frac{\boldsymbol{\mathcal{P}}_{kin}}{\sqrt{m^2 + \boldsymbol{\mathcal{P}}_{kin}^2}},\ \frac{d\zeta}{dx^0} = 0,\ \ \zeta = \pm 1,$$

$\boldsymbol{\mathcal{P}}^{kin} = \left(\mathcal{P}_i^{kin}\right)$. Thus, the classical theory describes both particle and antiparticles with charges ζq. One can prove that for independent variables $\boldsymbol{\eta} = (x^k, p_k, \zeta)$ equations of motion are canonical with an effective Hamiltonian $\mathcal{H}_{eff}$

$$\dot{\boldsymbol{\eta}} = \{\boldsymbol{\eta}, \mathcal{H}_{eff}\},\ \mathcal{H}_{eff} = \left[\zeta q A_0(x) + \omega\right]_{x^0 = \zeta\tau}\ . \tag{6}$$

Commutation relations for the operators $\hat{X}^k, \hat{P}_k, \hat{\zeta}$, which correspond to the variables x^k, p_k, ζ, we define according to their Poisson brackets, and we assume the operator $\hat{\zeta}$ to have the eigenvalues $\zeta = \pm 1$ by analogy with the classical theory. Thus, nonzero commutators are: $[\hat{X}^k, \hat{P}_j] = i\hbar\delta_j^k$ and $\hat{\zeta}^2 = 1$. As a state space we select one R, whose elements $\boldsymbol{\Psi} \in R$ are $\mathbf{x}$-dependent four-component columns ($\mathbf{x} = x^i$)

$$\mathbf{\Psi}=\begin{pmatrix}\Psi_{+1}(\mathbf{x})\\ \Psi_{-1}(\mathbf{x})\end{pmatrix},\ \Psi_{\zeta}(\mathbf{x})=\begin{pmatrix}\chi_{\zeta}(\mathbf{x})\\ \varphi_{\zeta}(\mathbf{x})\end{pmatrix},\ \zeta=\pm 1\,. \tag{7}$$

The inner product in R is defined as follows:

$$\begin{aligned}&(\mathbf{\Psi},\mathbf{\Psi}')=\left(\Psi_{+1},\Psi'_{+1}\right)+\left(\Psi'_{-1},\Psi_{-1}\right),\\ &(\Psi,\Psi')=\int\overline{\Psi}(\mathbf{x})\Psi'(\mathbf{x})d\mathbf{x}=\int\left[\chi^{*}(\mathbf{x})\varphi'(\mathbf{x})+\varphi^{*}(\mathbf{x})\chi'(\mathbf{x})\right]d\mathbf{x},\ \overline{\Psi}=\Psi^{+}\sigma_{1}.\end{aligned} \tag{8}$$

We seek basic operators in block-diagonal form, $\hat{\zeta}=\mathrm{bdiag}\,(I,-I)\,,\ \hat{X}^{k}=x^{k}\mathbf{I},\ \hat{P}_{k}=\hat{p}_{k}\mathbf{I},\ \ \hat{p}_{k}=-i\hbar\partial_{k}$, where I and $\mathbf{I}$ are 2×2 and 4×4 unit matrices respectively. A quantum Hamiltonian $\hat{H}_{\tau}$, which defines the evolution in τ, is constructing using its classical analog $\mathcal{H}_{eff}$,

$$\begin{aligned}&\hat{H}_{\tau}=\hat{\zeta}q\hat{A}_{0}+\hat{\Omega},\ \hat{\Omega}=\mathrm{bdiag}\left(\hat{\omega}|_{x^{0}=\tau}\ ,\hat{\omega}|_{x^{0}=-\tau}\right),\ \hat{\omega}=\begin{pmatrix}0&M\\ G&0\end{pmatrix},\\ &M=-\left[\hat{p}_{k}+qA_{k}\right]\sqrt{-g}g^{kj}\left[\hat{p}_{j}+qA_{j}\right]+m^{2}\sqrt{-g},\ G=\frac{g_{00}}{\sqrt{-g}}.\end{aligned} \tag{9}$$

The operator $\hat{A}_{0}=\mathrm{bdiag}\left(A_{0}|_{x^{0}=\tau}\,I,\ A_{0}|_{x^{0}=-\tau}\,I\right)$ is related to the classical quantity $A_{0}|_{x^{0}=\zeta\tau}$, and $\hat{\Omega}$ is related to the classical quantity $\omega|_{x^{0}=\zeta\tau}$. Indeed, $\hat{\Omega}^{2}=\mathrm{bdiag}\left(MG|_{x^{0}=\tau}\,I,\ \ GM|_{x^{0}=-\tau}\,I\right)$ corresponds (in classical limit) to square of the classical quantity $\omega|_{x^{0}=\zeta\tau}$. Quantum states evolute in time τ in accordance with the Schrödinger equation $i\hbar\partial_{\tau}\mathbf{\Psi}(\tau)=\hat{H}_{\tau}\mathbf{\Psi}(\tau)$, where the columns $\Psi_{\zeta}(\tau,\mathbf{x})$, and the functions $\varphi_{\zeta}(\tau,\mathbf{x})$, $\chi_{\zeta}(\tau,\mathbf{x})$ from (7) depend now on τ. As before we believe that $x^{0}=\zeta\tau$ may be treated as physical time and reformulate the evolution in its terms. At the same time we pass to another representation of state vectors.

$$\begin{aligned}&\mathbf{\Psi}(x^{0})=\begin{pmatrix}\Psi(x)\\ \Psi^{c}(x)\end{pmatrix},\ \Psi(x)=\begin{pmatrix}\chi(x)\\ \varphi(x)\end{pmatrix},\ \ \Psi^{c}(x)=\begin{pmatrix}\chi^{c}(x)\\ \varphi^{c}(x)\end{pmatrix},\\ &\Psi(x)=\Psi_{+1}(x^{0},\mathbf{x}),\ \Psi^{c}(x)=\Psi_{-1}^{*}(-x^{0},\mathbf{x}),\ \ x=\left(x^{0},\mathbf{x}\right)\ .\end{aligned} \tag{10}$$

The inner product of two states $\mathbf{\Psi}(x^{0})$ and $\mathbf{\Psi}'(x^{0})$ in such a representation takes the form

$$(\mathbf{\Psi},\mathbf{\Psi}')=(\Psi,\Psi')+\left(\Psi^{c},\Psi^{c\prime}\right), \tag{11}$$

where (Ψ,Ψ') is given by (8). In this representation the operators $\hat{\zeta}$ and $\hat{X}^k$ retain their form, whereas the Schrödinger equation changes

$$
\begin{aligned}
& i\hbar\partial_0\boldsymbol{\Psi}(x^0)=\hat{H}_{x^0}\boldsymbol{\Psi}(x^0),\ \hat{H}_{x^0}=\mathrm{bdiag}\left(\hat{h}(x^0),\hat{h}^c(x^0)\right)\,, \\
& \hat{h}(x^0)=qA_0I+\hat{\omega},\ \hat{h}^c(x^0)=\hat{h}(x^0)\Big|_{q\to -q}=-\left[\sigma_3\hat{h}(x^0)\sigma_3\right]^*. \qquad (12)
\end{aligned}
$$

In accordance to our interpretation $\hat{\zeta}$ is charge sign operator. Let $\boldsymbol{\Psi}_\zeta$ be states with a definite charge (ζq), $\hat{\zeta}\boldsymbol{\Psi}_\zeta=\zeta\boldsymbol{\Psi}_\zeta$. It is easily to see that states $\boldsymbol{\Psi}_{+1}$ with the charge q have $\Psi^c=0$. Then the equation (12) reads $i\hbar\partial_0\Psi=\hat{h}(x^0)\Psi$. In fact it is Klein-Gordon equation (KGE) for the charge q in first order form. It reproduces exactly the covariant KGE for the scalar field $\varphi(x)$ with the charge q,

$$
\left[\frac{1}{\sqrt{-g}}\left(i\hbar\partial_\mu-qA_\mu\right)\sqrt{-g}g^{\mu\nu}\left(i\hbar\partial_\nu-qA_\nu\right)-m^2\right]\varphi=0\,,\ \left(\chi=\sqrt{-g}g^{00}\left(i\partial_0-qA_0\right)\varphi\right)\,.
$$

States $\boldsymbol{\Psi}_{-1}$ with charge $-q$ have $\Psi=0$. In this case the equation (12) reads $i\hbar\partial_0\Psi^c=\hat{h}^c(x^0)\Psi^c$, with the Hamiltonian $\hat{h}^c(x^0)$, i.e. the KGE for the charge $-q$. The inner product (11) between two solutions with different charges is zero. For two solutions with charges q it takes the form of KGE scalar product for the case of the charge q. For two solutions with charges $-q$ the inner product (11) is expressed via KGE scalar product for the case of the charge $-q$. The Schrödinger equation (12) is totally charge invariant.

The eigenvalue problems for the Hamiltonians $\hat{h}$ and $\hat{h}^c$ in time independent external backgrounds

$$
\begin{aligned}
& \hat{h}\psi_{\varkappa,n}=\epsilon_{\varkappa,n}\psi_{\varkappa,n}\,,\ \ (\psi_{\varkappa,n},\psi_{\varkappa',n'})=\varkappa\delta_{\varkappa,\varkappa'}\delta_{n,n'}\,,\ \varkappa,\varkappa'=\pm\,, \\
& \hat{h}^c\psi^c_{\varkappa,n}=\epsilon^c_{\varkappa,n}\psi^c_{\varkappa,n}\,,\ \ \left(\psi^c_{k,n}\,,\psi^c_{\varkappa',n'}\right)=\varkappa\delta_{\varkappa,\varkappa'}\delta_{n,n'},\ \psi^c_{\varkappa,n}=-\sigma_3\psi^*_{-\varkappa,n}\,,\ \epsilon^c_{\varkappa,n}=-\epsilon_{-\varkappa,n}\,, \qquad (13)
\end{aligned}
$$

solve the eigenvalue problem of the Hamiltonian (12):

$$
\begin{aligned}
& \hat{H}_{x^0}\boldsymbol{\Psi}=E\boldsymbol{\Psi}\,,\ \boldsymbol{\Psi}=\left(\boldsymbol{\Psi}_{\varkappa,n}\,;\ \boldsymbol{\Psi}^c_{\varkappa,n}\right),\ E=\left(\epsilon_{\varkappa,n}\,;\,\epsilon^c_{\varkappa,n}\right)\,, \\
& \boldsymbol{\Psi}_{\varkappa,n}=\begin{pmatrix}\psi_{\varkappa,n}(\mathbf{x})\\ 0\end{pmatrix},\ \boldsymbol{\Psi}^c_{\varkappa,n}=\begin{pmatrix}0\\ \psi^c_{\varkappa,n}(\mathbf{x})\end{pmatrix},\ (\boldsymbol{\Psi},\boldsymbol{\Psi}^c)=0 \\
& \left(\boldsymbol{\Psi}^c_{\varkappa,n},\boldsymbol{\Psi}^c_{\varkappa',m}\right)=\left(\boldsymbol{\Psi}_{\varkappa,n},\boldsymbol{\Psi}_{\varkappa',m}\right)=\varkappa\delta_{\varkappa\varkappa'}\delta_{nm}\,,\ \varkappa=\pm\,. \qquad (14)
\end{aligned}
$$

On the Fig.1 we show typical spectra (one can keep in mind e.g. external Coulomb field):

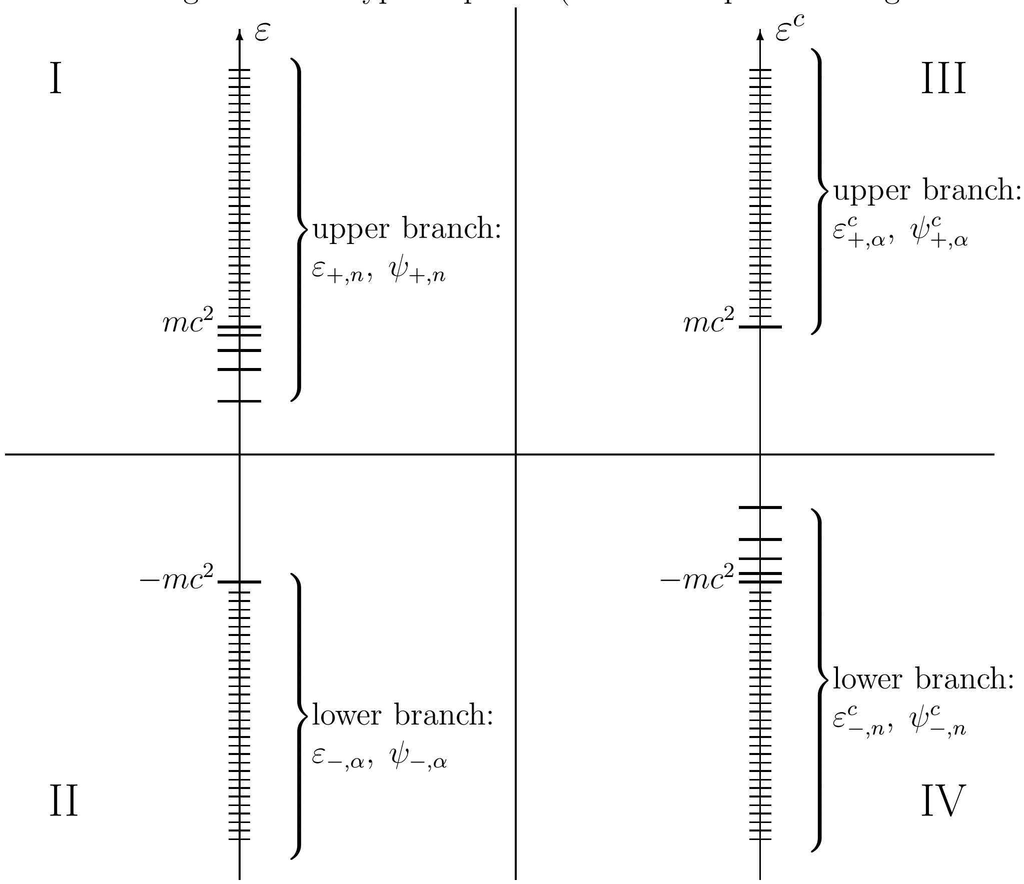

Fig.1. Energy spectra of KGE with charges q and $-q$; I and II - spectrum of $\hat{h}$, III and IV - spectrum of $\hat{h}^c$.

Let us compare results of the first quantization with one-particle sector of the corresponding QFT. In course of second quantization the the classical field (10) becomes operator $\hat{\Psi}(x)$ ($\hat{\Psi}^c = -(\hat{\Psi}^+\sigma_3)^T,\ \hat{\overline{\Psi}} = \hat{\Psi}^+\sigma_1$),

$$[\hat{\Psi}(x), \hat{\overline{\Psi}}(y)]_{x^0=y^0} = \hbar\delta(\mathbf{x}-\mathbf{y})\ ,\ i\hbar\partial_0\hat{\Psi}(x) = \hat{h}(x^0)\hat{\Psi}(x)\ ,\ i\hbar\partial_0\hat{\Psi}^c(x) = \hat{h}^c(x^0)\hat{\Psi}^c(x). \quad (15)$$

In external backgrounds, which do not create particles from the vacuum, one may define subspaces invariant under the evolution with definite numbers of particles . Let us consider below only such backgrounds which do not depend also on time to simplify the demonstration. A generalization to arbitrary backgrounds, in which the vacuum remains stable, looks similar. One may decompose the operator $\hat{\Psi}(x)$ in the complete set $\psi_{\varkappa,n}$, then $\hat{\Psi}(x) = \sum_n \left[a_n\psi_{+,n}(x) + b_n^+\psi_{-,n}(x)\right],\ \ [a_n, a_m^+] = [b_n, b_m^+] = \delta_{nm},\ [a_n, a_m] = [b_n, b_m] = 0$.

Thus, we get two sets of annihilation and creation operators a_n, a_n^+ and b_n, b_n^+, one of particles with a charge q and another one of antiparticles with a charge $-q$. Indeed, the Hamiltonian $\hat{H}^{QFT}$ of the QFT, charge operator $\hat{Q}^{QFT}$ and particle number operator $\hat{N}$ read

$$\hat{H}^{QFT} = \hat{H}_R^{QFT} + E_0,\ \hat{H}_R^{QFT} = \sum_n \left[\epsilon_{+,n} a_n^+ a_n + \epsilon_{+,n}^c b_n^+ b_n\right], E_0 = -\sum_n \epsilon_{-,n}\,,$$
$$\hat{Q}^{QFT} = q\sum_n \left[a_n^+ a_n - b_n^+ b_n\right]\ ,\ \hat{N} = \sum_n \left[a_n^+ a_n + b_n^+ b_n\right]\ ,$$

where $\hat{H}_R^{QFT}$ is a renormalized Hamiltonian. The Hilbert space R^{QFT} of QFT is a Fock one. In the backgrounds under consideration each subspace R_{AB}^{QFT} of state vectors with the given number of particles A and antiparticles B is invariant under the time evolution. Now we are in position to demonstrate that the one-particle sector of the QFT may be formulated as a consistent relativistic quantum mechanics. We reduce the space R^{QFT} to a subspace of vectors which obey the condition $\hat{N}|\mathbf{\Psi}>=|\mathbf{\Psi}>$. It is the subspace $R^1 = R_{10}^{QFT} \oplus R_{01}^{QFT}$. We call R^1 one-particle sector of QFT. All state vectors from the one-particle sector have positive norms. The spectrum of the Hamiltonian $\hat{H}_R^{QFT}$ in the space R^1 reproduces exactly one-particle energy spectrum of QFT (it is situated on the areas **I** and **III** of the Fig.1.),

$$\hat{H}_R^{QFT}|\mathbf{\Psi}>= E^{QFT}|\mathbf{\Psi}>,\ |\mathbf{\Psi}>= \left(a_n^+|0>;\ b_n^+|0>\right)\ ,\ E^{QFT} = \left(\epsilon_{+,n}\ ;\ \epsilon_{+,n}^c\right)\ . \tag{16}$$

The dynamics of the one-particle sector may be formulated in a coordinate representation, which is an analog of coordinate representation in nonrelativistic quantum mechanics. Let us consider time-dependent states $|\mathbf{\Psi}(x^0)>$ from the subspace R^1. One may describe these states in the coordinate representation by four columns

$$\mathbf{\Psi}(x^0) = \begin{pmatrix} \Psi(x) \\ \Psi^c(x) \end{pmatrix},\ \Psi(x) =< 0|\hat{\Psi}(x)|\mathbf{\Psi}(0)>,\ \Psi^c(x) =< 0|\hat{\Psi}^c(x)|\mathbf{\Psi}(0)>, \tag{17}$$

where $\Psi(x)$ and $\Psi^c(x)$ have the form (10). The QFT inner product reduces in this case to the inner product (11). One may find expressions for the basic operators in the coordinate representation in the one-particle sector. In particular, the Hamiltonian, the Schrödinger equation and charge operator $\hat{Q}^{QFT}$ are

$$\hat{H}_R^{QFT} \to \hat{H} = \mathrm{bdiag}\left(\hat{h}\,,\,\hat{h}^c\right)\;,\; i\hbar\partial_0\boldsymbol{\Psi}(x^0) = \hat{H}\boldsymbol{\Psi}(x^0),\; \hat{Q}^{QFT} \to \hat{Q} = q\hat{\zeta}\,. \tag{18}$$

We meet, in fact, all the quantum mechanical constructions in the case under consideration. The eigenvalue problem (16) in the coordinate representation has the form (14), however the states $\boldsymbol{\Psi}_{-,n}$ and $\boldsymbol{\Psi}^c_{-,\alpha}$ are absent. It reproduces the spectrum (16) situated on the areas **I** and **III** of Fig.1. According to superselection rules physical states are only those, which obey the condition $\hat{Q}^{QFT}\boldsymbol{\Psi}_\zeta = \hat{\zeta}q\boldsymbol{\Psi}_\zeta = \zeta q\boldsymbol{\Psi}_\zeta\,,\quad \zeta = \pm 1$. This condition defines a physical subspace $R^1_{ph} = R^{QFT}_{10} \cup R^{QFT}_{01}$ from the one-particle sector R^1. Due to the structure of the operator $\hat{\zeta}$, the states $\boldsymbol{\Psi}_{+1}$ contain only the upper half of components, whereas, ones $\boldsymbol{\Psi}_{-1}$ contain only the lower half of components. One may see that the complete set $\boldsymbol{\Psi}_{+,n}$ and $\boldsymbol{\Psi}^c_{+,\alpha}$ consists only of physical vectors.

Returning to the first quantization, we may see that under certain restrictions our quantum mechanics coincides literally with the one-particle sector of QFT. These restrictions are related only to an appropriate definition of the Hilbert space of the quantum mechanics. Indeed, all other constructions in the quantum mechanics and in the one-particle sector of the QFT in the coordinate representation coincide. Consider again the eigenvalue problem (14) for the quantum mechanical Hamiltonian in the space R. Its spectrum is wider than one of the QFT Hamiltonian in the space R^1. To get the same spectrum as in QFT, we need to eliminate the vectors $\boldsymbol{\Psi}_{-,n}$ and $\boldsymbol{\Psi}^c_{-,\alpha}$. We may define an analog of the space R^1 as a linear envelop of the vectors $\boldsymbol{\Psi}_{+,n}$ and $\boldsymbol{\Psi}^c_{+,\alpha}$ only. This space does not contain negative norm vectors. The spectrum of the Hamiltonian (12) in such defined space coincides with one of the QFT Hamiltonian in the one-particle sector. Reducing R^1 to R^1_{ph}, we get literal coincidence between both theories. One may think that the reduction of the space R of the quantum mechanics to the space R^1 is necessary only in the first quantization, thus an equivalence between the first and the second quantization is not complete. However, the same procedure is present in the second quantization. Indeed, besides the vectors (16) one could consider hypothetically the following possibilities to form one-particle states:

$$1)\; a_n^+|0\rangle_1,\; b_n|0\rangle_1,\; (a_n|0>_1 = b_n^+|0>_1 = 0);$$

$$2)\ a_n|0\rangle_2,\ b_n|0\rangle_2\,,\ (a_n^+|0>_2=b_n^+|0>_2=0)\,;$$

$$3)\ a_n|0\rangle_3,\ b_n^+|0\rangle_3,\ (a_n^+|0>_3=b_n|0>_3=0).$$

The states from the group 1) reproduce the usual spectrum of the KGE which is situated on the areas **I** and **II** (see Fig.1). The states from the group 2) reproduce the spectrum which is situated on the areas **IV** and **II**. The states from the group 3) reproduce the spectrum which is situated on the areas **IV** and **III**. These states are eliminated from the state space of the quantum field theory.

Thus, we see that the first quantization of classical actions of the relativistic particle leads to relativistic quantum mechanics, which is consistent to the same extent as corresponding quantum field theory in the one-particle sector. Such quantum mechanics describes charged particles of both signs (particles and antiparticles), and reproduces correctly their energy spectra without infinite number of negative energy levels. No negative vector norms need to be used in the corresponding Hilbert space. There is also an important analogy with the second quantization. Both in first and second quantizations we start with actions with a fixed charge and in course of the quantizations we get charge symmetric theories where particles and antiparticles are present on the same foot. It is also important to stress that the first quantization and its comparison with one-particle sector of the quantum field theory provides a very simple solution for the well-known old problem: how to construct a consistent quantum mechanics on the base of a relativistic wave equation? The solution is very simple, instead to try to use the lower branch of the spectrum (area **II** on Fig.1) one has to unite particle and antiparticle in one multiplet on the base of Schrödinger equation (12). Then the area **III** appears naturally, and areas **II** and **IV** have to be eliminated.

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